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COMPUTER-BASED DEVELOPMENT OF STRESS FIELDS

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Abstract

Computer-aided approaches for the efficient development of suitable stress fields are useful tools for the structural design of concrete structures. The paper describes a new procedure that is used within a previously proposed approach for the automatic generation of strut-and-tie models. Once the geometry of the strut-and-tie model is known, the stress fields can be created. A new method is also proposed to generate stress fields characterized by rectangular compression fields and nodes in a pseudo-hydrostatic stress state.

Keywords

Stress fields, strut-and-tie, concrete design, topological optimization

1 Introduction

In recent years, the use of strut-and-tie models and stress fields has become increasingly popular, since this approach is an efficient tool to describe the behavior of reinforced concrete structures, as demonstrated also by its adoption by some design codes. While finding the proper strut-and-tie model is straightforward in a variety of simple cases, it is more time consuming in many actual structures, where a specific expertise is necessary.
According to the lower bound theorem of the theory of plasticity, an infinite number of strut-and-tie models are possible to describe the internal forces within a given reinforced concrete structure. Reducing this number to a selected set of strut-and-tie models that are acceptable for both the serviceability and the ultimate limit states remains a difficult task. Since the compressive stresses in the structure need to be checked, compression fields must be derived from the strut-and-tie model.
This paper presents the first results of a research program that is focused on the improvement of an already-available approach to automatically generate strut-and-tie models and on the development of a new method to obtain stress fields with pseudo-hydrostatic state of stresses in all nodes.

2 Strut and tie models

The distribution of the resultants of the internal forces in concrete structures can be represented by strut-and-tie models. Previously called truss models, they were initially proposed by Ritter and Mörsch (fig. 1). Considerable contributions to the investigation on strut-and-tie models were made by Schlaich (Schlaich et al 1987). Strut-and-tie models are an idealization of the behavior of cracked reinforced concrete. By reducing the continuum to a discrete strut-and-tie model, a simpler approach to concrete and reinforced concrete modeling is made possible. Since concrete structures bear the applied loads according to the

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way they are reinforced, an infinite number of strut and tie models are possible. All of them need to satisfy equilibrium and to respect yield criteria.

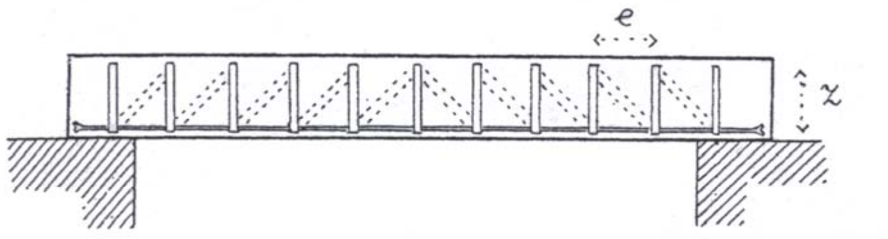
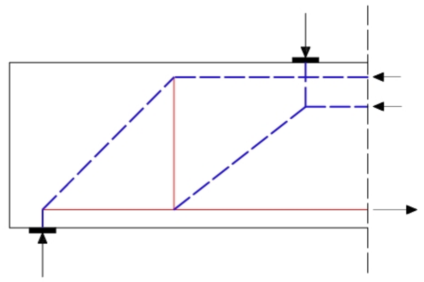


Figure 1 Ritter truss model (Ritter 1899)

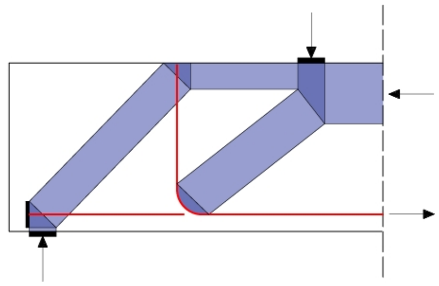
Some of these strut-and-tie models are not suitable because they would require high plastic deformations and force redistributions that are incompatible with the limited deformation capacity of concrete. Besides this problem, choosing an adequate strut-and-tie model for the ultimate limit state does not necessarily lead to a satisfactory behavior at the serviceability limit state. Using a strut-and-tie with a geometry that follows the elastic flow of the internal forces in the uncracked state leads to solutions that have a more satisfactory behavior in service. This approach, however, is not always practical, because it would often require the reinforcement to be placed in diagonal directions with complex shapes.

2.1 Evolution from strut-and-tie models toward stress fields

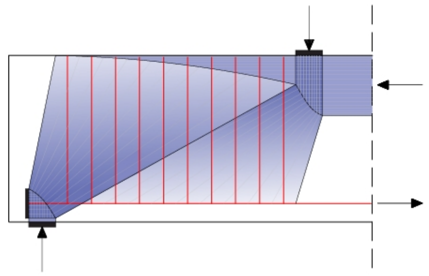
The main result obtained with the strut-and-tie models is the location and the amount of the required reinforcement. To check concrete failure, compression fields need to be used (Muttoni et al 1996, Nielsen 1999).
Compression fields occupy a sizable part of the volume of any reinforced concrete element. Their width results from the compression in the strut divided by the effective concrete strength and by the thickness of the element. On this basis, concentrated compression fields can be created (fig. 2b). The geometry of each compression field is a parallelogram. The compression fields cross each other in the nodes where tension fields may be present as well.



a) Strut and tie model



b) Concentrated stress field



c) Distributed stress field

Figure 2 Strut and tie model and corresponding stress fields

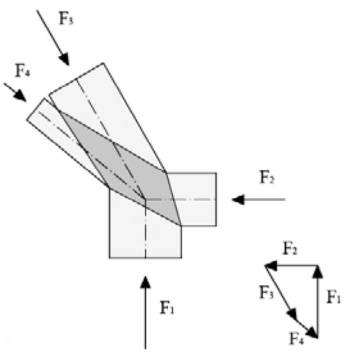
In concentrated stress fields, only concentrated reinforcement is included. In actual R/C however, a part of the reinforcement is distributed and the compressive stresses are diffused in the available concrete volume. Figure 2c shows a distributed stress field, which describes the behavior of the concrete element more realistically than the discrete stress field of figure 2b.

2.2 Nodal zones of stress fields

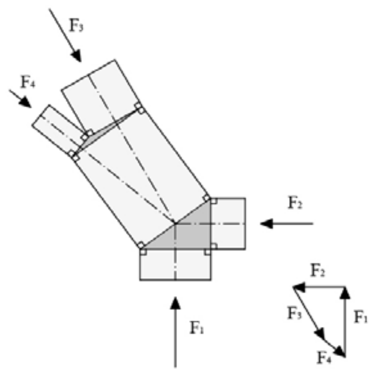
The nodes are the zones where of tension and/or compression fields joint or cross each other. They are regions where the internal forces are deviated. The nodes are essential parts of a stress field and need to be checked to ensure a proper behavior at the ultimate limit state. In this section, the generation of the nodes including only compression fields is discussed.

Figure 3a shows a node with the intersection of four compression fields. The stress in each compression field is constant and is equal to the design concrete strength f_{cd}. Because the lines of action of the compression fields cross in one point, the corners of the node are defined by the intersection of the edges of the compression fields. If the edges of the node are not perpendicular to the incoming compression field, shearing stresses are present along the edges and in the node. Checks on the load-carrying capacity of the nodes can be made by using a failure criterion, on the basis of the principal stresses (Marti 1985).

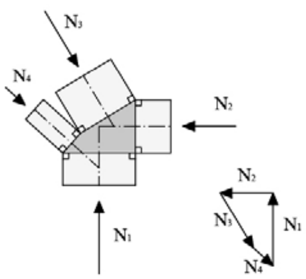
If all edges of a node are perpendicular to the direction of the incoming compression field, the state of stress within the node itself is pseudo-hydrostatic with the stresses being $\sigma_1 = \sigma_2 = f_{cd}$ and $\sigma_3 = 0$. Since the strength of concrete under bi-axial pseudo-hydrostatic compression is larger than f_{cd} (Kupfer 1964) and since no shear stress is present, there is actually no need to perform additional strength checks in this type of node (fig. 3c). Consequently a stress field model with all the nodes in a pseudo-hydrostatic state of stresses greatly simplifies the design checks.



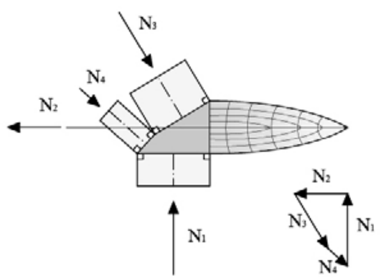
(a) node defined by the intersection of four compression fields



(b) pseudo-hydrostatic node according to Anderheggen and Schlaich, (1990)



(c) pseudo-hydrostatic node



(d) pseudo-hydrostatic node with introduction of a tensile force into the concrete

Figure 3 Different nodes connecting 4 compression fields

3 Automatic generation of strut and ties models

Finding a proper strut-and-tie model is a time consuming process for most of the actual structures and automatic procedures are badly needed. A procedure initially proposed for steel trusses (Dorn et al 1964) was adapted to reinforced concrete (Bendsoe et al 1994, Biondini et al 1996). It consists of the generation of an initial strut-and-tie model where a series of predefined joints are interconnected by bars. This transforms the continuum into an n-times statically indeterminate strut-and-tie model (fig. 4b). The position of the joints can be selected by the designer or can be performed automatically.

3.1 Stiffness-based procedure

A new stiffness-based procedure to select a suitable strut-and-tie model from the initial one is proposed in the following. Some elements of the initial strut-and-tie model are not as efficient in transmitting the forces as others and can thus be removed. Starting from the same stiffness in all the elements of the initial strut-and-tie model, an elastic solution can be obtained from

$$Ku = f \tag{1}$$

where K is the stiffness matrix, u is the displacement vector of the nodes and f is the vector of the applied forces. Using the steel strength for tension members and the concrete strength for compression members, new sectional dimensions for all members are obtained after the first iteration. The stiffnesses obtained in this way are used as an input for the next iteration. The assumed stiffness of i^{th} element in the j^{th} iteration can be written as:

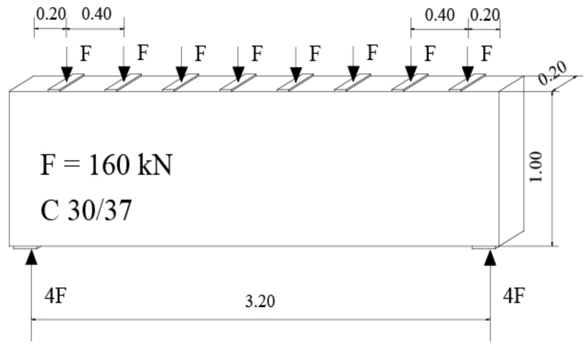
$$K_{i,j} = \beta_i \frac{E_d N_{i,j-1}}{L_i f_d} \tag{2}$$

where $N_{i,j-1}$ is the force in the i^{th} element at the $j\text{-}1^{\text{th}}$ iteration, E_d and f_d are the Young moduli and the strength of steel and concrete. The “practicality factor” β_i is applied to the tension bars that run in any direction, where the reinforcement cannot be placed for practical reasons; it can take values between zero and one (Ali and White 2001). Its function is to penalize any reinforcement placed in undesirable directions during the iterative process.

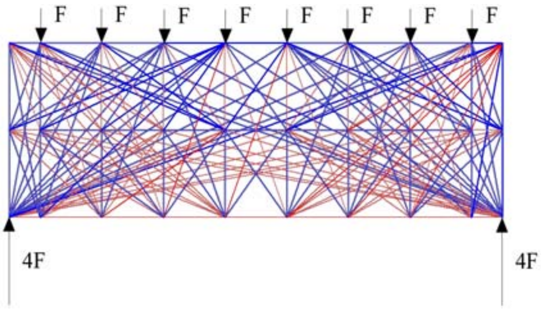
The above-described approach is only a practical way to identify the most effective elements for transmitting the forces and to gradually remove the least effective elements. With regard to thus point, a procedure that takes in consideration the actual rigidity of each compression element is under development.

3.2 Application to a deep beam

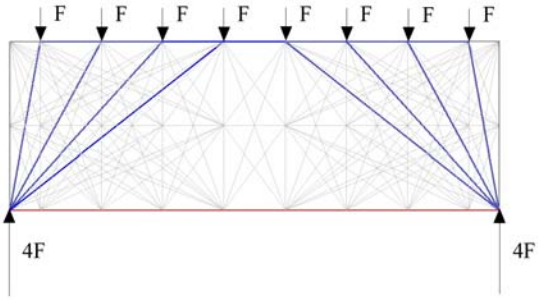
Figure 4a shows a deep beam with a rectangular cross section subjected to a series of concentrated forces. The initial strut-and-tie model is shown in figure 4b. After the iterative process, the number of elements in the strut-and-tie model is significantly reduced and the well-know strut-and-tie model of figure 4c is obtained. It consists of struts that directly bear the loads to the supports. The horizontal component of the force in the inclined struts is carried by horizontal struts and ties. Since compression struts are stiffer than tension ties, the final model includes predominantly compression elements. Figure 4d shows the results of the same beam subjected to asymmetrical loads. It shows that a direct support with too small an inclination is automatically replaced during the procedure by an indirect support with two compression diagonals and one vertical tie, because this configuration is stiffer. The practicality factor β_i used for the above mentioned cases is equal to 0.1 for inclined ties and to 1.0 for all other members.



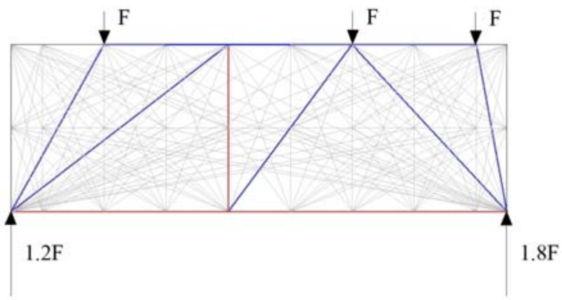
a) Deep beam: properties and applied loads



b) Initial strut-and-tie model



c) Strut-and-tie model with direct support action



d) Strut-and-tie model with partially indirect support action

Figure 4 Strut-and-tie models for a deep beam

4 Generation of the geometry of the stress fields

Once a suitable strut-and-tie model has been developed, it is necessary to derive the stress field to check concrete stresses. Compression fields are often idealized as discrete compression members. It is desirable that all the nodes of the stress field be in a pseudo-hydrostatic state of stresses. To obtain this condition, the direction and the position of some members (fig. 3a) should be slightly changed (fig. 3c). This requires the forces in the system to change as well, which is something to be taken in account and to be achieved through a suitable iterative process.

A procedure for obtaining a node in a pseudo-hydrostatic state of stress is to combine the struts two-by-two to form a resultant strut and a pseudo-hydrostatic node (Anderheggen and Schlaich 1990). Repeating this procedure for all the compression fields pertaining to a node leads to the correct solution (fig. 3b). An alternative procedure was developed within this research project. According to this procedure, a single node with three or more incoming members or forces is obtained. In this case, the axes of the compression fields do not meet in a single point, this being always the case when four or more members joint together (fig. 3c).

4.1 Geometry based procedure

This section presents the procedure used to obtain a stress field in which all the nodes are in a state of pseudo-hydrostatic stresses, starting from a known strut-and-tie model.

First, compression fields are generated by assuming a constant concrete compressive strength f_{cd}. The nodes are created by the intersection of the compression fields. The stress field for which all the nodes are in a pseudo-hydrostatic state of stresses can be found iteratively by modifying the geometry. As was previously observed, a node is in pseudo-hydrostatic state of stress when all the compression fields are rectangular and perpendicular to the edges of the node. Starting from the initial configuration of each node (fig. 5b), a suitable procedure has to be formulated in order to modify the location and the shape of the edges, by transforming each compression field into a rectangle.

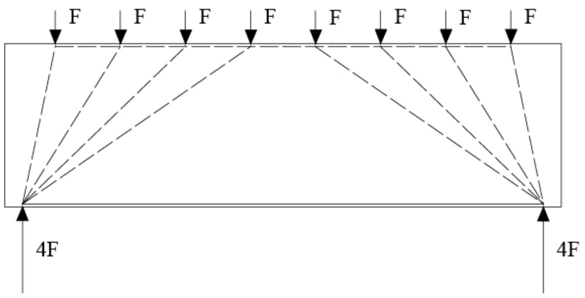
This can be achieved by formulating a suitable objective function based on the difference between each strut angle and the right angle (to the square) for all compression fields:

$$\sum_{i=1}^{n}\sum_{j=1}^{4}\left(\alpha_j - \frac{\pi}{2}\right)^2 = 0 : x_k , y_k = const \qquad (3)$$

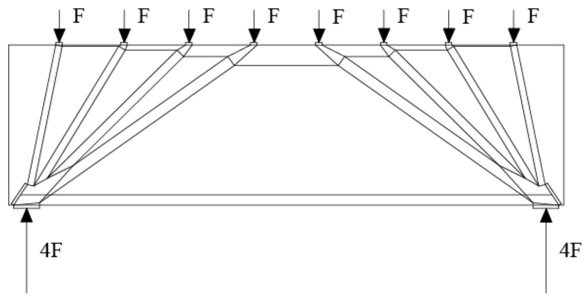
where n is the number of the compression fields in the model, α_j refers to the four angles of each compression field, x_k and y_k are the positions of the points that need to remain fixed in the structure. Fixed points (x_k , y_k) are typically related to applied forces or reactions at supports. All the other points of intersections of the compression fields are considered free to change their position to obtain a rectangular shape for all compression fields. Additional constraints can be introduced: for example, all points need to remain within the structure.
Within the algorithm, the tension fields acting on a node are replaced with equivalent compressive-stress distribution acting on the same node. Figure 3d show how the tensile force is converted into a compressive force. In this procedure, the shape of the tension fields also needs to be rectangular.

4.2 Example of application

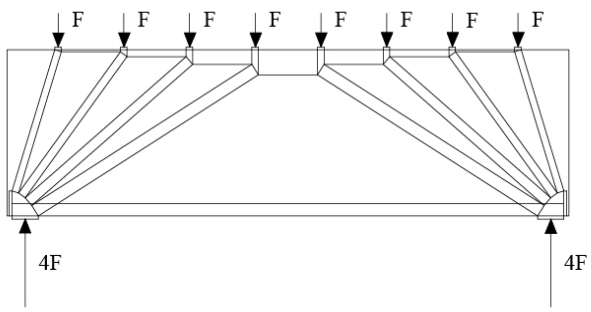
The strut-and-tie model obtained earlier for a deep beam is shown in figure 5a. An initial stress field is created by assuming an equal stress f_{cd} in all compression fields (fig. 5b). In this model, the nodes obtained by the intersection of the compression fields are generally not in a pseudo-hydrostatic state of stresses. Hence a change in the position and inclination of the compression fields is necessary by using the above-described procedure, and by keeping without changing the points where the loads and the reactions are introduced, the shape of the stress field is modified. The final stress field is shown in figure 5c.



a) Strut and tie model



b) Initial stress field



c) Stress field with pseudo-hydrostatic nodes

Figure 5 Stress field for a deep beam

5 Conclusions

This paper presents an automatic stiffness-based procedure for the generation of strut-and-tie models. Starting from an initial strut-and-tie model with a large number of members, the least effective members are successively removed by means of an iterative process. The proposed procedure automatically leads to reasonable and well known strut-and-tie models.
Once a suitable strut-and-tie model has been developed, it has to be transformed into a stress field. This is done by means of an algorithm that minimizes a function of the form of the compression fields joining in each node. In such a way, the nodes are in a pseudo-hydrostatic state of stress that makes the check of the stresses in the concrete very simple. It is also shown that this procedure leading to the automatic generation of stress fields with all the nodes in a pseudo-hydrostatic state of stress can be applied to any strut-and-tie model.

6 Further work

Further work will be concentrated on the topological optimization of strut and tie models. The procedure described in this paper will be improved to allow the introduction of additional constraints related for instance to the ductility demand, to the serviceability requirements and to the introduction of local forces.

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